



## 13+ Scholarship Examinations 2019

### MATHEMATICS I

**Seventy-five minutes (plus five minutes reading time)**

*Use the reading time wisely; gain an overview of the paper and start to think how you will answer the questions.*

*Do as many questions as you can (clearly numbered) on the lined paper provided. Clearly name each sheet used.*

*The questions are not of equal length or mark allocation. Move on quickly if stuck; you are not expected to finish.*

*You are expected to use a calculator where appropriate, but you must show **full and clear working**, diagrams and arguments wherever you can. Marks will be awarded for method as well as answers: merely writing down an answer might score very few marks.*

*Complete solutions are preferable to fragments. You can sometimes, however, manage to complete later parts of questions, even if you have failed to answer the earlier sections.*

*This paper has fourteen questions.*

1 (a) William Shakespeare (the playwright) is thought to have possessed a vocabulary as large as 65000 words. How long would it take him to say aloud each of these words once? Make your assumptions clear and give the answer time in appropriate units.

(b) Marlowe claims: “if I count aloud from one to one billion I will use only thirty-one different number words”.

Show that he is correct, if we ignore connecting words such as “and” (n.b. you do not need to list all the words).

2 Two trains set out simultaneously from London and Edinburgh. They travel towards each other down tracks which are parallel.

- The Edinburgh to London train travels at a constant 50 mph.
- The London to Edinburgh train travels at a constant 90 mph.

How far apart will the trains be one hour before they pass each other?

3 In this question you may wish to use the following facts:

Speed of light	299 792 458 m / s
1 foot	12 inches
1 inch	2.54 cm
1 nanosecond	One billionth of a second

Here is the recently-issued picture of the supermassive black hole M87\* which is about 73 million light-years away from Earth.



(a) The actual black hole is about 100 billion kilometres across. How long would it take me to travel that distance at the speed of light? (Of course, I would not manage to cross the actual black hole!) Give your answer in appropriate units.

(b) My friend claims: “if I travel at the speed of light I will cover a distance of one foot in a nanosecond”.

Showing your working carefully, demonstrate that this claim is almost correct.

4 Greta tells me I can save the planet by doing some green improvements to my house

- Improvement A will save 25% of whatever energy I am currently using
- Improvement B will save 45% of whatever energy I am currently using
- Improvement C will save 30% of whatever energy I am currently using

On Monday I do A, on Tuesday I do B, and on Wednesday I do C (these improvements are **permanent** so do not just last one day).

What will my overall compounded percentage energy saving be?

5 At King's the Headmaster and the Beadle are standing in the Shirley Hall. At the front of the hall is the stage.

When the Headmaster stands on the stage, and the Beadle stands on the floor below, the top of the Headmaster's head is 150 cm above the top of the Beadle's bowler hat.

When the Beadle stands on the stage, and the Headmaster stands on the floor below, the top of the Beadle's bowler hat is 110 cm above the top of the Headmaster's head.

How tall is the stage?

[Hint: you may wish to try drawing diagrams to help you with this question. Show your working carefully.]

6 Here is a tweet which was quoted in the national newspapers earlier this year.



Ben Stephens @stephens\_ben

Fascinating little life hack, for doing percentages:  $x\%$  of  $y = y\%$  of  $x$  So, for example, if you needed to work out  $4\%$  of  $75$  in your head, just flip it and do  $75\%$  of  $4$ , which is easier.

3:21 AM - 3 Mar 2019

- (a) Explain why this claim is always true, i.e. that  $x\%$  of  $y = y\%$  of  $x$
- (b) Give an example where this method would not really help.

7 Carruthers sits nine scholarship papers. His average score is 92. If his highest score (96) and his lowest are not counted then the average is 94.

What is Carruthers' lowest score?

8 Solve the following for  $x$ :

(a)

$$\frac{2}{5x-1} = -3$$

(b)

$$\frac{2x}{7} + \frac{4}{9} = 1$$

(c)

$$3x + 5(8 + 7x) = 41 - 13(2 - x)$$

9 Barney goes into his scholarship examination carrying a bag of pens.

The bag contains **some** black pens.

It also contains one more green pen than there are black pens.

It also contains one more blue pen than there are green pens.

Barney chooses one at random.

What is the probability that he picks a green pen?

You must show all working carefully, aiming to say something general rather than specific.

10 Another scholarship examination in a different school has two **compulsory** papers.

You are given that

- 80% of candidates pass paper one
- 70% of candidates pass paper two
- 55% of candidates pass both papers

You choose a candidate at random after the examinations have been sat.

I tell you that she failed paper one.

What is the probability that she passed paper two?

11 Three candidates are sitting a two-hour scholarship examination. They each spend their time either writing or thinking. From the start at twelve noon:

- A writes for exactly three minutes then thinks for exactly three minutes (then repeats this pattern for the rest of the examination).
- B writes for exactly four minutes then thinks for exactly four minutes (then repeats this pattern for the rest of the examination).
- C writes for exactly five minutes then thinks for exactly five minutes (then repeats this pattern for the rest of the examination).

- (a) When is the first time that all three are thinking simultaneously?
- (b) After the start, when is the next time that all three candidates will start writing at exactly the same moment?
- (c) What is the total time during which all three candidates are writing simultaneously?

12 At a school, a little like King's, there are 800 pupils in total.

- 30% of the Sixth Form like tackling questions on percentages
- 40% of the non-Sixth formers do not like doing percentages questions.

Considering the whole school, 50.5% of all the pupils do not like doing percentage questions.

What is the number of non-Sixth former pupils that like tackling questions about percentages?

13

Consider three scholarship candidates: Judith, Neil and Marc. They each support a different football team, one of Liverpool, Manchester City and Tottenham – but not necessarily in that order.

Only **one** of the following statements is true.

- Judith supports Liverpool
- Neil does not support Liverpool
- Marc does not support Tottenham

Work out which team each of them supports, making your reasoning clear.

14 Working alone, Andrew can do a task in ten hours.

Andrew works for four hours and then Ben joins, and then the task is completed two hours later.

How long would it take Ben working alone?

**END OF QUESTION PAPER**

# 13+ Scholarship Examinations

## May 2021

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(plus five minutes reading time)

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*Do as many questions as you can in the spaces provided. Use extra paper only if you have to and make sure you clearly name any sheets used.*

*The questions are not of equal length or mark allocation. Move on quickly if stuck; you are not expected to finish.*

*You are expected to use a calculator where appropriate, but you must show full and clear working, diagrams and arguments wherever you can. Marks will be awarded for method as well as answers: merely writing down an answer might score very few marks.*

*Complete solutions are preferable to fragments. You can sometimes, however, manage to complete later parts of questions, even if you have failed to answer the earlier sections.*

*This paper has twelve questions.*

# 1

In the Thirteenth Century Cistercian monks invented a numbering system in which they could write any number from 1 to 9999 using a single symbol. The basic system is shown below together with five example numbers written down. The symbols are always drawn about a central line which has no value on its own.

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1000	2000	3000	4000	5000	6000	7000	8000	9000
1993	4723	6859	7085	9433				

Using the five examples shown above to help you figure out the patterns, draw carefully the symbol to depict

(a) 2021

(b) 3537

(c) 7852

## 2

In this question **honest coppers** are police officers who always tell the truth. **Bent coppers** always lie. All the characters mentioned in this question are police officers.

(a) Ted meets Buckells and asks him: “are you an honest copper?”

Buckells mutters an answer which Ted cannot hear, so Ted says:

“I couldn’t hear that. Did you say you were an honest copper?”

Buckells replies: “No, I did not; I said I was a bent copper”.

What is Buckells? Explain your answer.

(b) The next day, Ted meets Jo, Kate and Steve and asks Jo: “Are you a bent copper?”

When Jo answers, someone coughs loudly and Ted cannot hear her answer.

Ted asks Kate: “What did Jo say?”

Kate replies: “Jo said she is a bent copper.”

Then Steve says: “Don’t believe what Kate says – she is lying.”

What can Ted deduce about Steve and Kate? Explain your answer.

## 3

Emma was born at midnight on 21 May 1990. At what date and time will she be one billion seconds old?

[Ignore leap years, leap seconds etc. in this question.]



## 4

In this question assume there are at least eighteen scholarship candidates for mathematics.

In the mathematics examination there are  $n$  candidates, and their mean mark is nine.

If a subset of seventeen candidates has a mean score of fifteen, work out an algebraic expression for the mean mark of the **remaining** candidates (in terms of  $n$ ).

## 5

Prefix	Meaning
Milli-	One thousandth
Micro-	One millionth
Nano-	One billionth

(a) Feynman claims: ‘there are “hyperfactorial 5” milliseconds in one day’.

Showing your working carefully, calculate whether this claim is true or not.

$$[\text{Hyperfactorial } 5 \text{ is } 5^5 \times 4^4 \times 3^3 \times 2^2 \times 1^1]$$

(b) Albert claims ‘there are pi seconds in a nano-century’.

Work out the **percentage error** in this claim (use the  $\pi$  button on your calculator).

# 6

Solve the following for  $x$ :

(a)  $7(6x + 1) = -56$

(b)  $19 - 5(4 + x) = 9x + \frac{1}{3}(8 - x)$

# 7

The Japanese runner Shizo Kanakuri fell asleep while taking a break during the 1912 Olympic marathon in Stockholm. In 1967 he was invited to return and finish the race, and was given a final time of 54 years, 8 months, 6 days, 5 hours, 32 minutes, and 20.3 seconds.

The length of a marathon race is 42.195 km.

[Ignore leap years etc. again in this question.]

- (a) Work out his average speed, giving your answer in appropriate units.

- (b) Comment how this compares with some of the slowest creatures on Earth (see below):

<b>Creature</b>	<b>Speed</b>
Sea Anemone	1 centimetre (.04 inch) per hour.
Three-toed Sloth	two kilometres (1.2 miles) per hour
Galapagos Tortoise	1 foot per minute

Another runner, Indian Arvind Morarbhai Pandya, once ran **backwards** from John O' Groats to Land's End, UK in 26 days and 7 hours.

- (c) If he listened (streaming on Spotify, say) to the classic hit song “Keep on Running” (which lasts 2 minutes 45 seconds) continuously for all that time, and if Spotify pays the artist \$0.0032 per song streamed, how much money would his listening earn for the artist? Give your answer in dollars and cents.

## 8

Suppose 130 candidates take another scholarship exam marked out of 100 (no half marks). No three score the same.

What is the smallest number of pairs who can gain the same mark? Explain your answer.

## 9

Froome and Bernal go cycling; they start together and ride towards a mountain. Froome is a bit slow.

Bernal cycles at 20 mph until he reaches the halfway point, when the road becomes very steep and his speed immediately halves. When Bernal reaches the top, he turns straightaway and carefully cycles down the mountain (the same route) at 15 mph, meeting Froome at the halfway point. What was Froome's **exact** mean speed until they meet?

## 10

In another mathematics examination, consisting of two papers, there is a mean score of 78 on paper 1 and a mean of 65 on paper 2. Different numbers of candidates sit each paper, in the ratio 8:5.

What is the mean score of all the papers, taken together?

# 11

Only 5% of scholar candidates are **mathmos**. You would like to be a mathmo. There is a test for this.

If you are a mathmo then, sitting the test, there is a 95% chance the paper will tell you you are one.

If you are not a mathmo then the test will show that to be the case 80% of the time.

- (a) You take a test and get a positive result for being a mathmo. How reliable is this result? For your working you might, for instance, imagine 1000 candidates taking the test. Explain your answer carefully.

- (b) Of all those testing positive first time, we try and make sure by giving a second test, with identical percentages to those given above. You test positive a second time. Are you happy now with the result? Once again, show your working carefully.

One in ten thousand people is **eccentric**. For some reason, you do not want to be eccentric. There is a test for this too.

If you are eccentric then the test will tell you 99% of the time.

Very rarely, the test will tell you a false positive and say you are eccentric when you are not, but this only happens in one in a thousand tests.

- (c) You test positive for eccentricity. Should you be worried? Once again, show your working carefully.

# 12

In another scholarship mathematics examination, there are the following options:

Papers

- A1      A2
- B1      B2
- C1      C2
- D1      D2

The rules are:

- You must take at least two papers and can sit up to and including all eight.
- You cannot take a paper 2 without doing the paper 1 before it.

[In answering these question parts, do not try to list them all but count them in organised fashion.]

(a) How many ways are there of sitting eight papers?

(b) Explain why there are ten ways of choosing two papers.

(c) Explain carefully why there are 76 ways in total of choosing a set of papers.